A MATHEMATICAL APPROACH FOR COST AND SCHEDULE RISK ATTRIBUTION

NASA 2014 COST SYMPOSIUM

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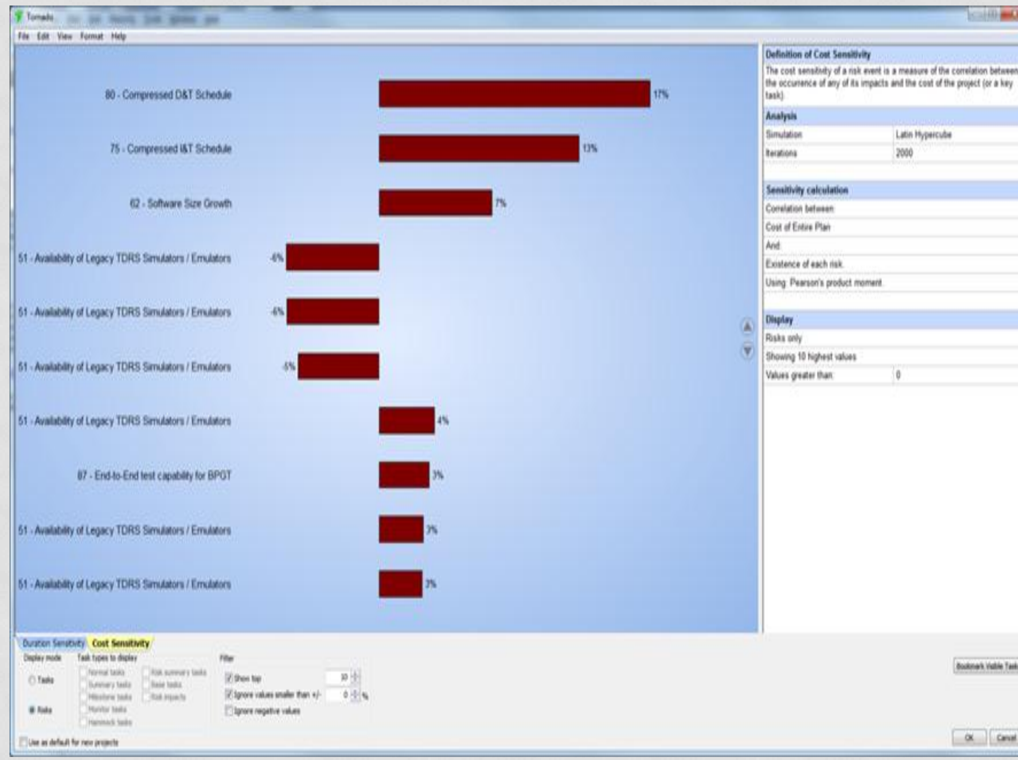
MOTIVATION

- There are many cost/schedule risk tools that allow analyst to perform more complex simulations, and that is a good thing.
- We have a good understanding, from the current tools, an overall risks impact on cost and schedule.
- Confidence Level and Joint Confidence Level analyses results are well understood, and are supported by various tools.
- One shortcoming for most of simulation tools is the individual risk's contribution to the overall project cost or schedule duration.
- There are tools that only hint at the "significance of contribution" through sensitivity analysis and Tornado charts. Some outputs are ambiguous and hard to understand what it means.
- For example, see Pertmaster tool

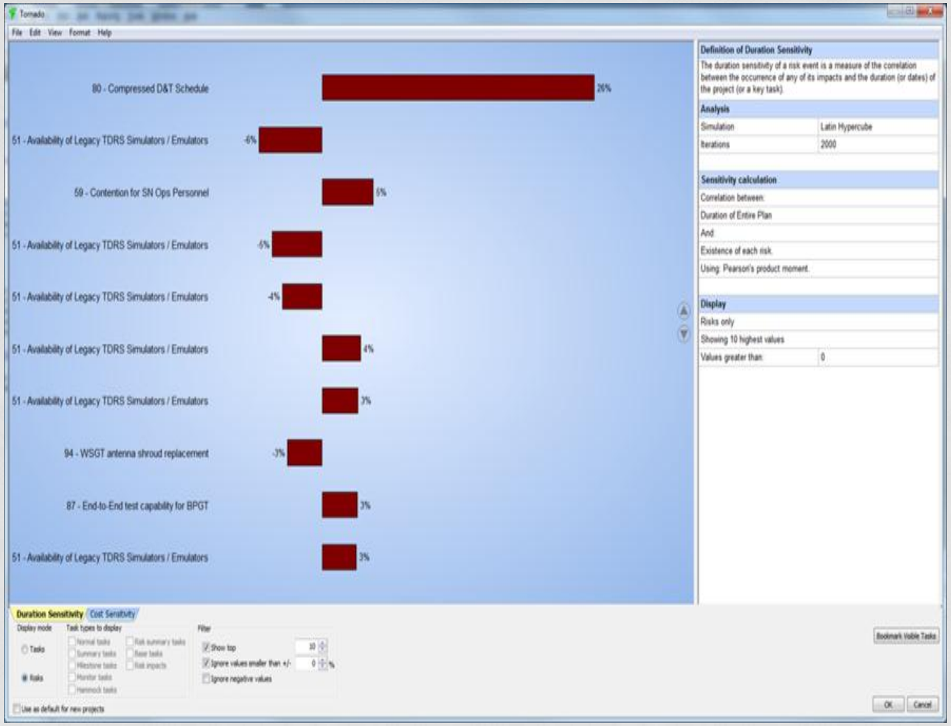
EXAMPLE
COST RISK SENSITIVITY

- The ***cost sensitivity*** of a task is a measure of the correlation between its cost and the cost of the project (or a key task or summary).
- What does that mean? And how do I use this information?



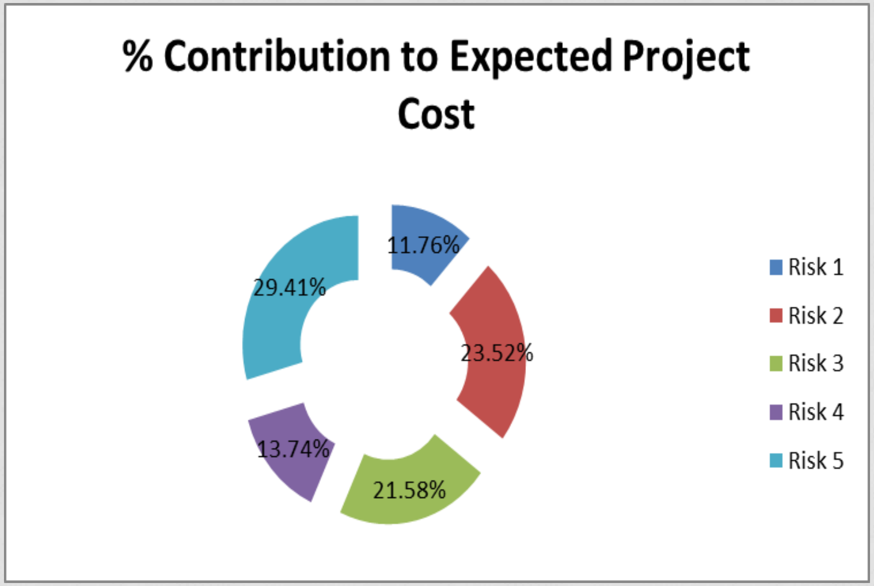
ANOTHER EXAMPLE
SCHEDULE RISK SENSITIVITY

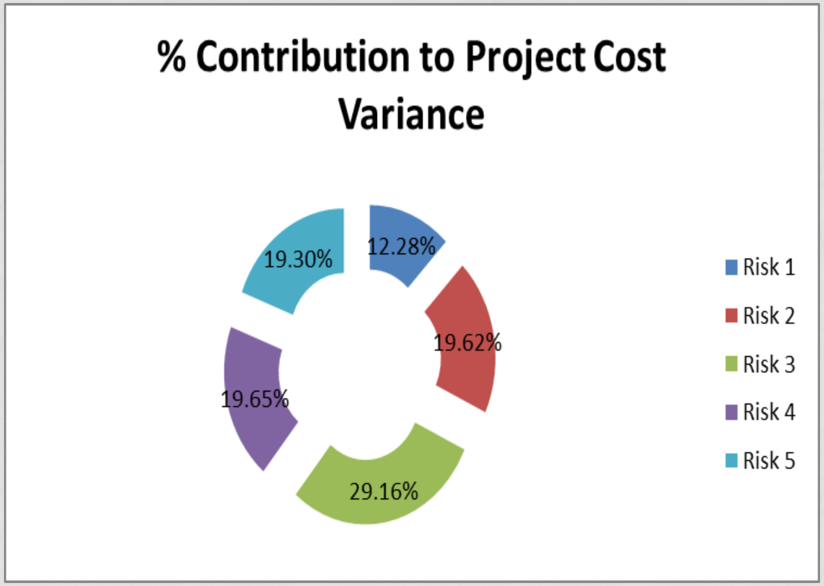
- The ***duration sensitivity*** of a risk event is a measure of the correlation between the occurrence of any of its impacts and the duration (or dates) of the project (or a key task).
- What does that mean? And how do I use this information?
- What does negative sign means? Does it mean higher risk will actually reduce my duration?



Correlation is not a good sensitivity measure, especially for schedule

A MORE CONCISE VIEW WOULD SHOW





Why can't we have some explicit measures like this?

HOW DO WE GET THERE?

- Borrowing a concept of “Portfolio” from financial industry
  - The main attributes of a portfolio of assets are its expected return and standard deviation. Financial industry defines risk by “volatility”, which is basically standard deviation.
  - Standard deviation defines the steepness of the S-Curve or “riskiness” of the estimate in the parlance of cost/schedule analysis as well.
  - The familiar formulas are:

    $r_p = \sum_{i=1}^{n} w_i r_i$

    $\sigma_p = \sqrt{w' \Sigma w}$

    r_i is the return of asset *i*

    w_i is the weight of asset *i* in the portfolio

    $\Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix}$ is the covariance matrix

    $w = [\, w_1, w_2, \ldots, w_n]$ is a vector of portfolio weights

    w' is the transpose of w.
    - Note that portfolio weights are not unique, for instance SP500 is market capitalization weighted, and DJ Industrial is price weighted

WHY CHOOSE THIS PORTFOLIO APPROACH?

- $\sigma_p = \sqrt{w'\Sigma w}$ is a homogeneous function of degree one
- The advantage of choosing σ_p as the risk measure is that now we can decompose risks as:

$$\sigma_p = w_1 \frac{\partial \sigma_p}{\partial w_1} + w_2 \frac{\partial \sigma_p}{\partial w_2} + \ldots\ldots\ldots + w_n \frac{\partial \sigma_p}{\partial w_n} \qquad \text{(Euler's Theorem)}$$

Note that

$MCR_1 = \frac{\partial \sigma_p}{\partial w_1}$ is defined as the marginal contribution to risk measure by risk #1

Then

$CR_1 = w_1 * MCR_1$ is the contribution to risk measure by risk #1,

and the total risk is the summation of each of the risk contribution CR_i

$\sigma_p = CR_1 + CR_2 + \ldots\ldots + CR_n$

So the percent contribution from each risk is

$PCR_i = \frac{CR_i}{\sigma_p}$

ANALOGOUS TERMS IN COST AND SCHEDULE RISKS

- Main attributes of interest in cost estimate and risks
  - Expected cost estimate (mean cost)
  - Cost estimate standard deviation (steepness of cost estimate S-Curve)
- Main attributes of interest in schedule risks
  - Expected project duration (translate to project schedule)
  - Schedule duration standard deviation (steepness of schedule S-Curve)
- These two attributes can be reframed in the portfolio sense

  $\mu_p = \sum_{i=1}^{n} \mu_i$, and

  $\sigma_p = \sqrt{w' \Sigma w}$

  where now we define $w_i = \frac{\mu_i}{\mu_p}$, and $\sum_{i=1}^{n} w_i = 1$
- The intuition here is that "**portfolio standard deviation is weighted by individual's mean**"
- This selection of weights is not unique but reasonable, just like SP500 and DJ Industrial

HERE IS THE MECHANICS OF CALCULATION

- **Derivation of MCR (some calculus and matrix algebra)**

$$\frac{\partial \sigma_p}{\partial \boldsymbol{w}} = \frac{\partial (\boldsymbol{w}' \boldsymbol{\Sigma} \boldsymbol{w})^{\frac{1}{2}}}{\partial \boldsymbol{w}} = (\boldsymbol{w}' \boldsymbol{\Sigma} \boldsymbol{w})^{\frac{-1}{2}} (\boldsymbol{\Sigma} \boldsymbol{w}) = \frac{\boldsymbol{\Sigma} \boldsymbol{w}}{(\boldsymbol{w}' \boldsymbol{\Sigma} \boldsymbol{w})^{\frac{1}{2}}} = \frac{\boldsymbol{\Sigma} \boldsymbol{w}}{\sigma_p}$$

So, $\frac{\partial \sigma_p}{\partial w_i} = \text{ith row of } = \frac{\boldsymbol{\Sigma} \boldsymbol{w}}{\sigma_p}$

- **Example for a portfolio of 2 Risks**

$$\sigma_p = \sqrt{w' \Sigma w}$$

$$\Sigma w = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} w_1 \sigma_1^2 + w_2 \sigma_{12} \\ w_2 \sigma_2^2 + w_1 \sigma_{12} \end{pmatrix}$$

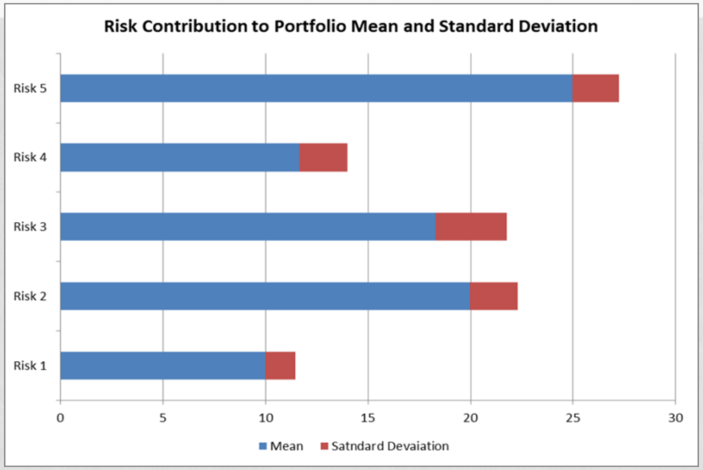
$$\frac{\Sigma w}{\sigma_p} = \begin{pmatrix} \frac{w_1 \sigma_1^2 + w_2 \sigma_{12}}{\sigma_p} \\ \frac{w_2 \sigma_2^2 + w_1 \sigma_{12}}{\sigma_p} \end{pmatrix} = \begin{pmatrix} MCR_1 \\ MCR_2 \end{pmatrix}$$

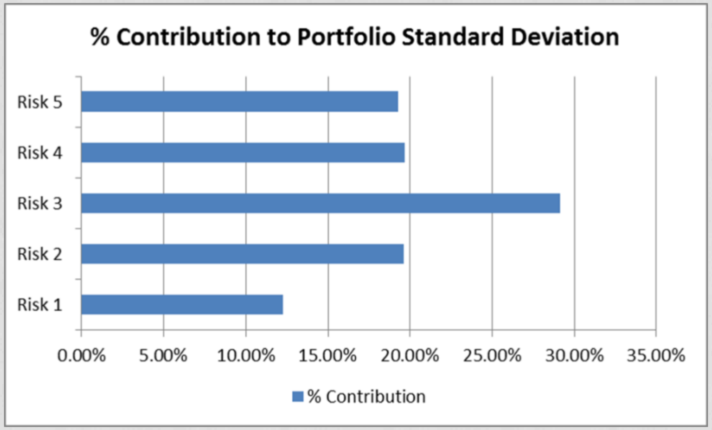
- $CR_1 = w_1 \, MCR_1 \;\; ; PCR_1 = \frac{CR_1}{\sigma_p} = \frac{w_1^2 \sigma_1^2 + w_1 w_2 \sigma_{12}}{\sigma_p^2}$
- $CR_2 = w_2 \, MCR_2 \;\; ; PCR_2 = \frac{CR_2}{\sigma_p} = \frac{w_2^2 \sigma_2^2 + w_1 w_2 \sigma_{12}}{\sigma_p^2}$
- It is obvious that $\sum_{i=1}^{n} PCR_i = 1$, the sum of "percent contribution to risks" equals 1.

SIMPLE EXAMPLES

- A portfolio of 5 risks, or a project with 5 subsystems.
- Assign a correlation of 0.5
- The mean cost is 84.41, and SD is 11.88

	Type	Mean	SD	W(i)	MCR(i)	CR(i)	PCR(i)
Risk 1	Lognormal	9.981	2.004	0.118	0.146	0.017	0.123
Risk 2	Lognormal	19.957	3.013	0.235	0.117	0.028	0.196
Risk 3	Triangular	18.312	4.236	0.216	0.189	0.041	0.292
Risk 4	Triangular	11.658	3.046	0.137	0.200	0.028	0.197
Risk 5	Normal	24.962	2.981	0.294	0.092	0.027	0.193
Portfolio		84.411	11.881	1.000		0.140	1.000



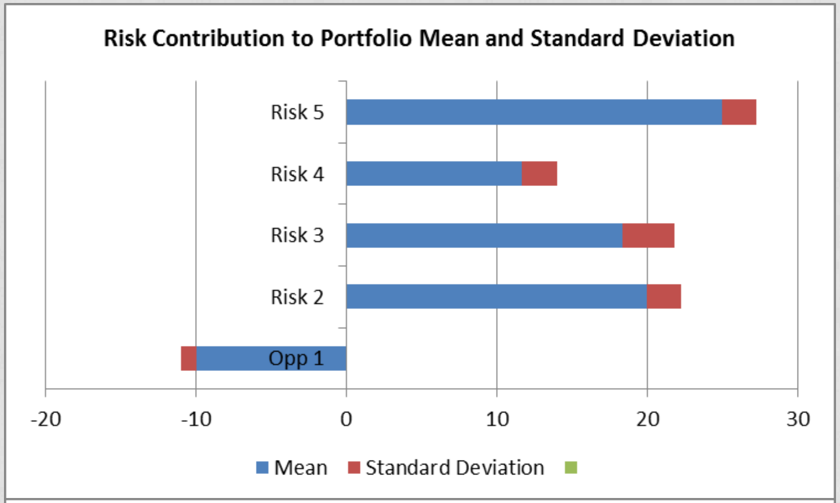


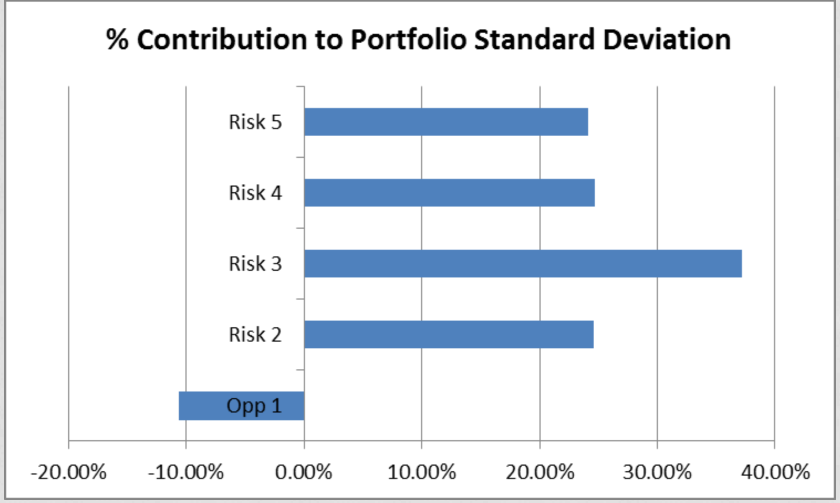
SIMPLE EXAMPLES WITH OPPORTUNITY

- A portfolio of 4 risks, and 1 opportunity
- The mean cost is 64.908, and SD is 9.376
- Notice that w_1 is now negative, indicating that it is an opportunity instead of risk

	Type	Mean	SD	W(i)	MCR(i)	CR(i)	PCR(i)
Opp 1	Lognormal	-9.981	2.004	-0.154	0.099	-0.015	-0.106
Risk 2	Lognormal	19.957	3.013	0.307	0.116	0.036	0.246
Risk 3	Triangular	18.312	4.236	0.282	0.190	0.054	0.372
Risk 4	Triangular	11.658	3.046	0.180	0.198	0.036	0.247
Risk 5	Normal	24.962	2.981	0.385	0.091	0.035	0.242
Portfolio		64.908	9.376	1.000		0.144	1.000

- So opportunity should reduce the mean and standard deviation, as we would expect.



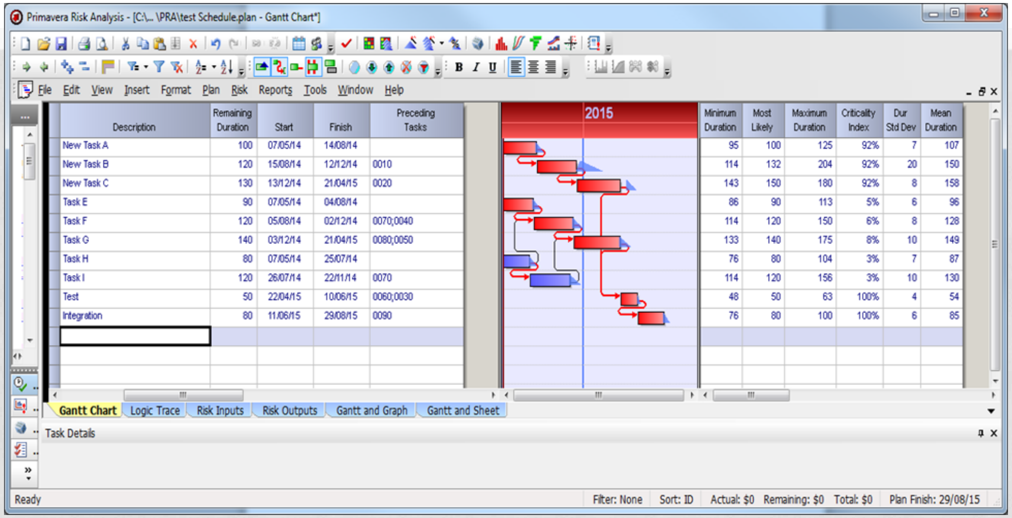


HOW TO EXTEND TO SCHEDULE RISK

- What is a portfolio in a schedule sense?
- How do we define this portfolio in a project with many tasks?
  - Main measure is project duration, driven by critical path.
  - Not every task contributes to critical path though all contributes to overall costs.
  - So a portfolio for schedule should only consists of tasks that are on, or potentially will be on critical path.
  - Make use of criticality index, a common output of many schedule tools, to define critical tasks.

SCHEDULE EXAMPLES (1) WITH TASK UNCERTAINTIES ONLY

- Unlike cost, not all tasks will contribute to project duration.
- Only the tasks with probability on the critical path will contribute to the expected project duration and standard deviation.
- We can conceive a portfolio of tasks with non zero criticality index.
- Comparing PertMaster outputs and calculated outputs using criticality index shows very proximate results.

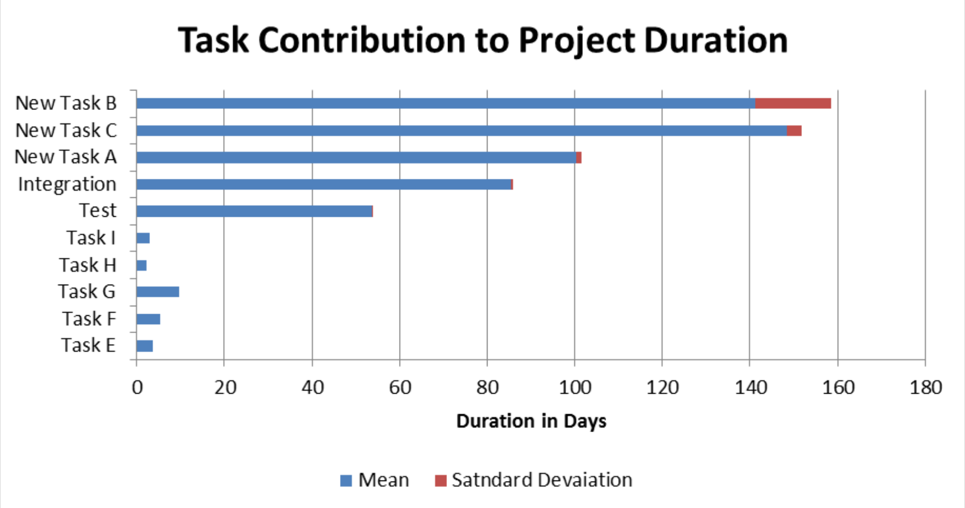


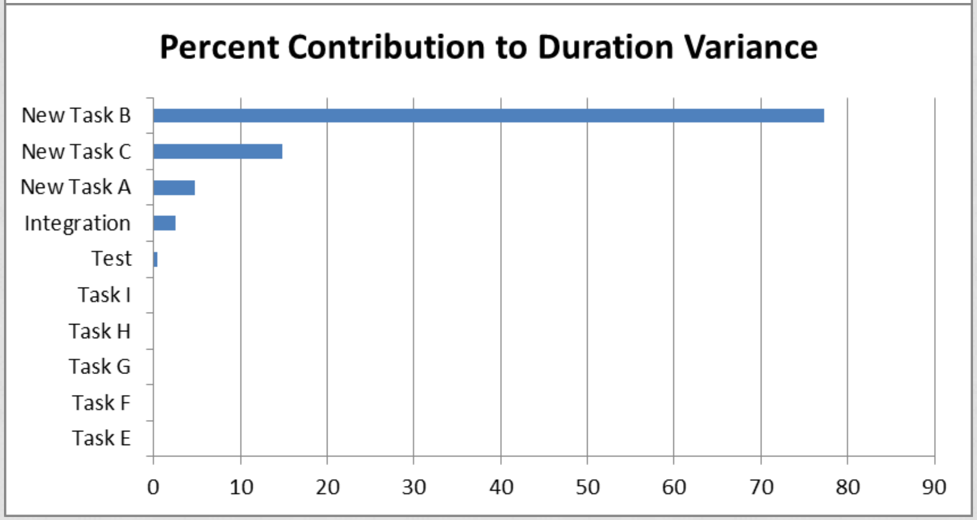
	L	ML	H	Cri_index	SD	Mean Duration	Mean* Cri_index	Sd* Cri_index
New Task A	95.00	100.00	125.00	94.10	6.93	106.67	100.38	6.52
New Task B	114.00	132.00	204.00	94.10	19.82	150.00	141.15	18.65
New Task C	143.00	150.00	180.00	94.10	8.40	157.66	148.36	7.90
Task E	86.00	90.00	113.00	3.90	6.32	96.33	3.76	0.25
Task F	114.00	120.00	150.00	4.20	8.25	128.00	5.38	0.35
Task G	133.00	140.00	175.00	6.40	9.56	149.33	9.56	0.61
Task H	76.00	80.00	104.00	2.60	6.55	86.67	2.25	0.17
Task I	114.00	120.00	156.00	2.20	9.64	130.00	2.86	0.21
Test	48.00	50.00	63.00	100.00	3.68	53.67	53.67	3.68
Integration	76.00	80.00	100.00	100.00	5.62	85.34	85.34	5.62
Portfolio					22.47	553.00	552.70	22.33
	Model output							
	Calculated							

SCHEDULE EXAMPLES (1) WITH TASK UNCERTAINTIES ONLY

- Applying the same technique to this portfolio, the following results were obtained.

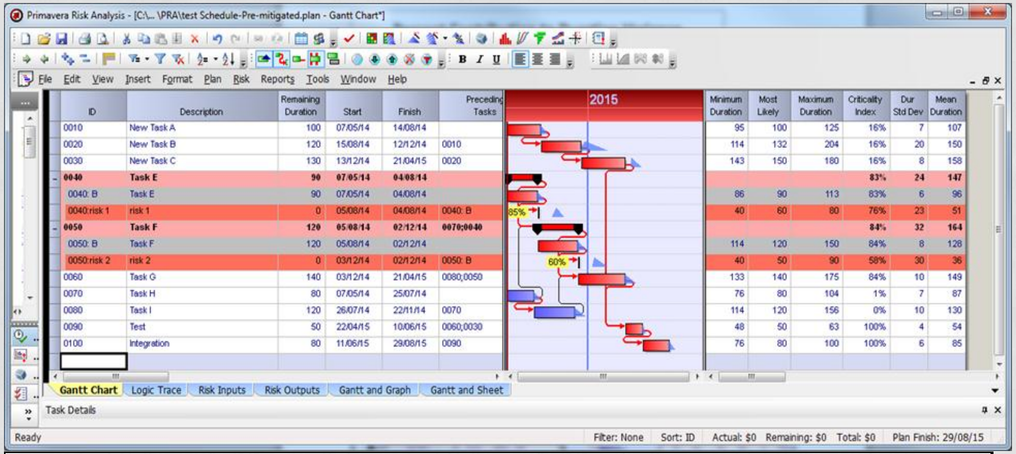
	Mean	SD	W(i)	MCR(i)	CR(i)	PCR(i)
New Task A	95.00	6.93	0.18	0.0634	0.0115	0.0478
New Task B	114.00	19.82	0.26	0.7299	0.1864	0.7733
New Task C	143.00	8.40	0.27	0.1336	0.0359	0.1488
Task E	86.00	6.32	0.01	0.0000	0.0000	0.0000
Task F	114.00	8.25	0.01	0.0000	0.0000	0.0000
Task G	133.00	9.56	0.02	0.0001	0.0000	0.0000
Task H	76.00	6.55	0.00	0.0000	0.0000	0.0000
Task I	114.00	9.64	0.01	0.0000	0.0000	0.0000
Test	48.00	3.68	0.10	0.0108	0.0010	0.0043
Integration	76.00	5.62	0.15	0.0401	0.0062	0.0257
Portfolio	553.00	22.47	1.0000		0.2410	0.9999





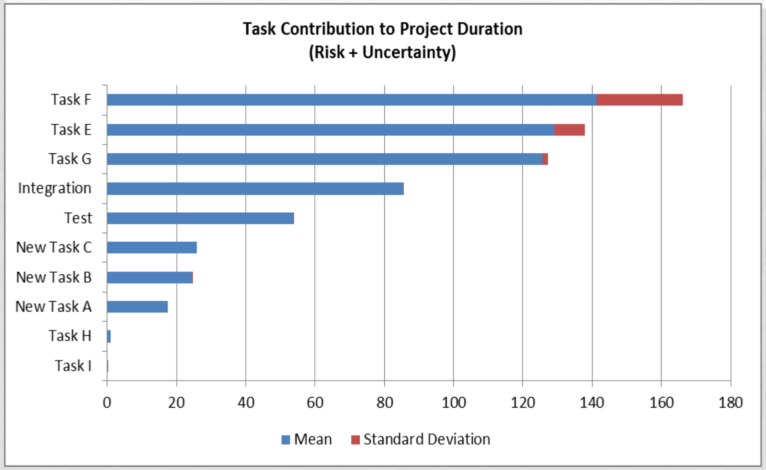
SCHEDULE EXAMPLES (2)
WITH TASK UNCERTAINTIES PLUS RISKS

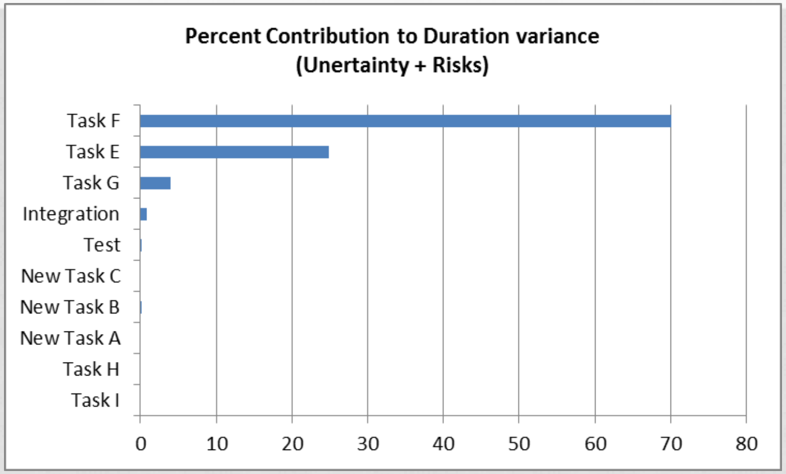
- In this case 2 discrete risks were added.
- Adding discrete risks changes the dynamics of the critical path.
- Discrete risks push Tasks E,F,G to be on the critical path.
- It is also important to note that discrete risks increases portfolio standard deviation substantially.
- For example, discrete risks increase expected duration by 9.2% but standard deviation by 59%.
- The increase in variance of discrete risks is due to binomial nature of probability of existence of risks.

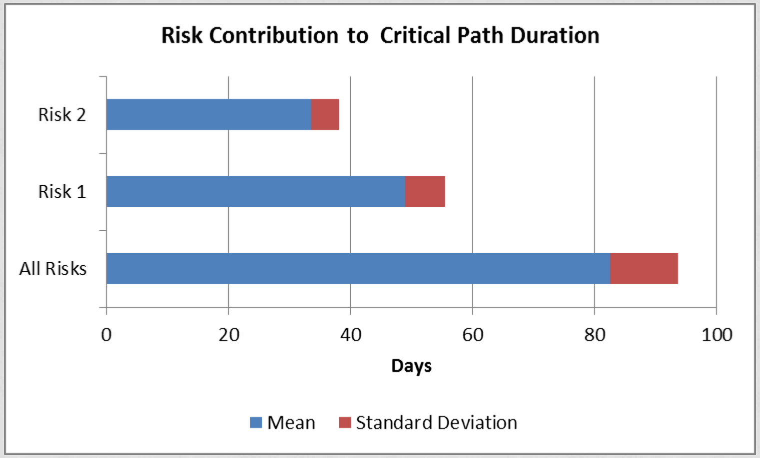


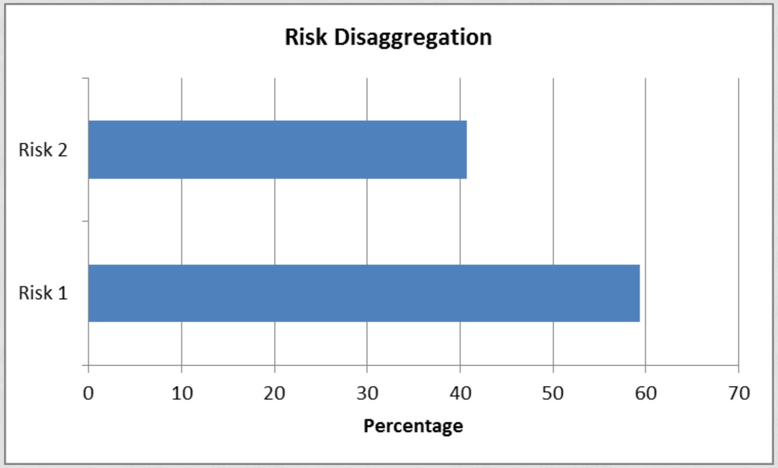
	L	ML	H	Cri_index	SD	Mean Duration	Mean* Cri_index	Sd* Cri_index
New Task A	95	100	125	16.35	6.93	106.67	17.44	1.13
New Task B	114	132	204	16.35	19.83	150	24.53	3.24
New Task C	143	150	180	16.35	8.4	157.67	25.78	1.37
Task E				83.16	23.67	147.37	0.00	19.68
Task E	86	90	113	83.16	6.32	96.33	80.11	5.26
risk 1	40	60	80	96.06	22.84	51.04	49.03	21.94
Task F				84.08	31.84	164.07	0.00	26.77
Task F	114	120	150	84.08	8.25	128	107.62	6.94
risk 2	40	50	90	93.25	30.68	36.07	33.64	28.61
Task G	133	140	175	84.21	9.56	149.33	125.75	8.05
Task H	76	80	104	1.18	6.55	86.67	1.02	0.08
Task I	114	120	156	0.13	9.64	130	0.17	0.01
Test	48	50	63	100.00	3.68	53.67	53.67	3.68
Integration	76	80	100	100.00	5.62	85.33	85.33	5.62
Portfolio						Model (MC)	604.00	35.50
						Calculated	604.08	35.04

SCHEDULE EXAMPLES (2) WITH TASK UNCERTAINTIES PLUS RISKS



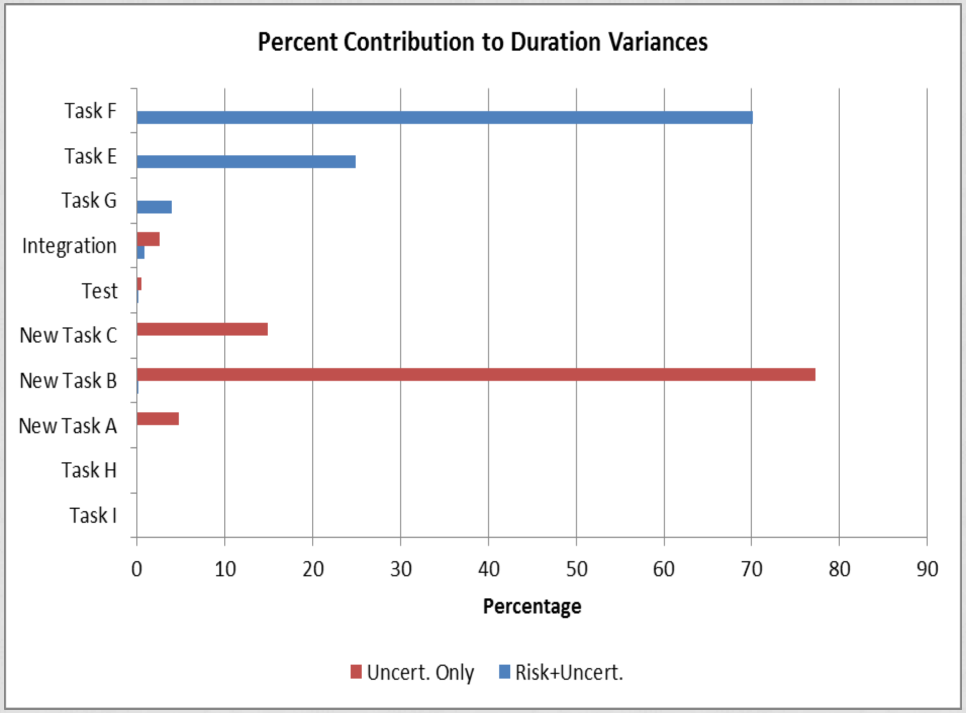






SCHEDULE EXAMPLES COMPARISON

- Given the myriad of data available, one can further compare or extract more useful information from the data.
- For example, this graph shows that discrete risks change the dynamics of the schedule substantially.
- This example shows also that schedule model is highly non-linear, so correlating and task with the project duration as in the case of “schedule sensitivity index” is not meaningful.



CONCLUSION AND FUTURE WORK

- A portfolio approach to risk attribution for cost and schedule risks, and the mathematical framework has been developed.
- This risk attribution methodology can be extended to include cost “opportunity” in reducing the expected cost and cost variance as one would expect.
- The same methodology can be extended to schedule risks by properly considering only the tasks that affect the critical path as a portfolio.
- This algorithm provides a more precise risk impact quantification and disaggregation so that each risk/uncertainty can be better quantified.
- The methodology is simple and can be incorporated easily into existing cost/schedule simulation tools using mainly matrix operations.
- This algorithm has not been tested for more complex risk topology such as multiple risks assigned to the same task, serial or parallel assignment of risks to the same task.
- Therefore, future work will consider this more complex topology.